Optimal foil shape for neutron time-of-flight measurements using elastic recoils

D. G. Hicks,[a)] C. K. Li, F. H. Séguin, J. A. Frenje, and R. D. Petrasso[b)]
Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

T. C. Sangster
Lawrence Livermore National Laboratory, Livermore, California 94550

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The basis for a time-of-flight neutron spectrometer for inertial confinement fusion (ICF) experiments using recoils from a shaped scattering foil is presented. It is shown that the number of elastic recoils can be substantially increased by utilizing a large scattering foil in the shape of an ellipsoid, with the curvature of the ellipsoid being determined by the mass of the recoil particle. This shape allows the time-of-flight dispersion — present originally in the neutrons — to be maintained in the recoils despite the large foil area. The feasibility of using this design on current ICF experiments is discussed. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1321002]

I. INTRODUCTION

With the progress towards higher yields and ρR's in inertial confinement fusion experiments, it has become important to develop new designs for neutron spectrometers to enable measurements of parameters such as fuel and shell ρR.[1,2] In particular, recoil spectrometers are of interest because the elastic scattering cross sections and the expected energy distributions are well defined. This has the important advantage that the measured signal is absolutely calibrated, allowing accurate measurements of neutron yield.

A typical recoil spectrometer design requires the dimensions of the scattering foil to be small. This allows the relationship between the observed recoil energy and the incident neutron energy to be accurately determined by the kinetics of elastic scattering, with minimal uncertainty about where in the scattering foil the recoil was produced. For time-of-flight detector systems, this ensures that the arrival time of the recoils can be used to infer the neutron energy, while the pulse height of the signal gives the fluence of neutrons. The problem with this approach is that, by using such a small scattering foil volume, the detected signal is also small.

Moran[3] attempted to mitigate this low-signal problem by using a proton-recoil generator in the shape of an annulus, centered on the line-of-sight between the target source and the time-of-flight detector. All parts of the annulus subtend the same angle at the detector and contribute recoils of the same energy. These recoils all travel the same path length from the target to the detector and thus the signal is still time-of-flight dispersed, i.e., neutrons of a given energy, interacting with different portions of the annulus, produce recoils which arrive at the detector at the same time. Despite this improvement, signal levels were still low. To increase the signal further requires the annulus be broadened to a shell-like structure which necessarily allows recoils scattered at different angles (and thus of different energies) to reach the detector.

The purpose of this discussion is to show that there is a precise, mathematically defined shape of the recoil foil which ensures that all recoils generated by neutrons of the same energy (the neutrons being time-of-flight dispersed) arrive at the detector at the same time, regardless of their scattering angle and energy. As will be shown, this shape is defined only if the recoils are heavier than protons. An analysis of the expected signal, spectrum, and signal-to-noise of a deuteron-recoil spectrometer of this kind will be performed. In its present form, a suggested detector would be a silicon PIN-diode operated in current-mode. Note that, in this discussion, it is the time-integrated neutron spectrum that is desired.

II. SCATTERING FOIL SHAPE FOR CONSTANT TIME-OF-FLIGHT

Consider the scattering of neutrons from a foil as shown in Fig. 1. If the system is far enough from the source, neutrons of a given energy arrive together in a ''wave front.'' Depending on the shape of the foil, neutrons from this front scatter at different times from various parts of the foil. The basis of this concept is that recoils generated at any point on the foil (by neutrons from this wave front) arrive at the detector at the same time.

Consider a planar wave front of neutrons arriving at a height $2h$ at time $t=0$. Recoils produced at point C arrive at the detector at time $t=2h/v_r^{\max}$, where $v_r^{\max}$ is the maximum recoil velocity, as produced by forward scattering. During this period, a neutron from the same wave front can travel on to point P in time t_1, produce a recoil at an angle θ, which then travels on to the detector in a time t_2. If the foil is shaped such that $t=t_1+t_2$, then

$$\frac{2h}{v_r^{\max}}=\frac{2h-y}{v_n}+\frac{\sqrt{x^2+y^2}}{v_r}, \qquad (1)$$

[a)]Present address: Lawrence Livermore National Laboratory, Livermore, CA 94550; electronic mail: hicks13@llnl.gov
[b)]Visiting Senior Scientist at the Laboratory for Laser Energetics, University of Rochester.

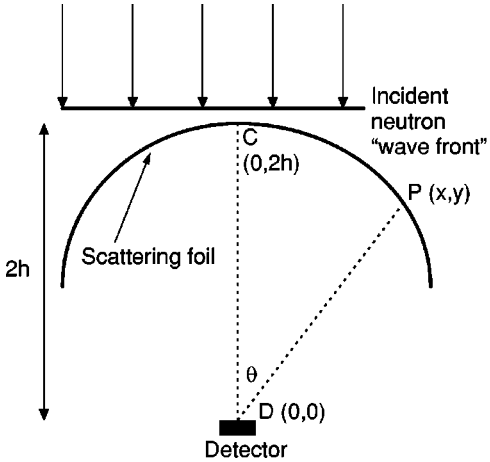

FIG. 1. Recoils produced at point P (x,y) must arrive at D (0,0) at the same time as recoils produced at point C $(0,2h)$ by the same planar wave front.

where v_r is the recoil velocity.

Now a particle, with mass number A, elastically scattered by a neutron, acquires a velocity, v_r, given by

$$v_r = v_n \zeta \cos\theta, \tag{2}$$

where $\zeta = 2/(1+A)$, θ is the angle of scattering with respect to the initial neutron direction, and v_n is the neutron velocity. From this it can be seen that $v_r^{\max} = v_n \zeta$.

Substituting this into Eq. (1), and using the relationship $\cos\theta = y/\sqrt{x^2+y^2}$ gives

$$\frac{x^2}{(1-\zeta)h^2} + \frac{(y-h)^2}{h^2} = 1. \tag{3}$$

This is simply the equation of an ellipse, centered at the point $(0,h)$, with major axis (in the y-direction) of length h and minor axis (in the x-direction) of length $h\sqrt{1-\zeta}$. In three dimensions, this shape is, of course, an ellipsoid.

Now for protons, $\zeta = 2/(1+A) = 1$, which makes the minor axis of the ellipse zero and Eq. (3) undefined. Thus it is impossible to construct a foil of this kind for use with proton recoils. The obvious alternative is to use deuterons, where $\zeta = \frac{2}{3}$. The shape of the deuteron foil is shown in Fig. 2 for $h = 0.5$. It is worthwhile noting that for very large A, where $\zeta \rightarrow 0$, the foil shape approaches a circle, or, in 3-D, a sphere.

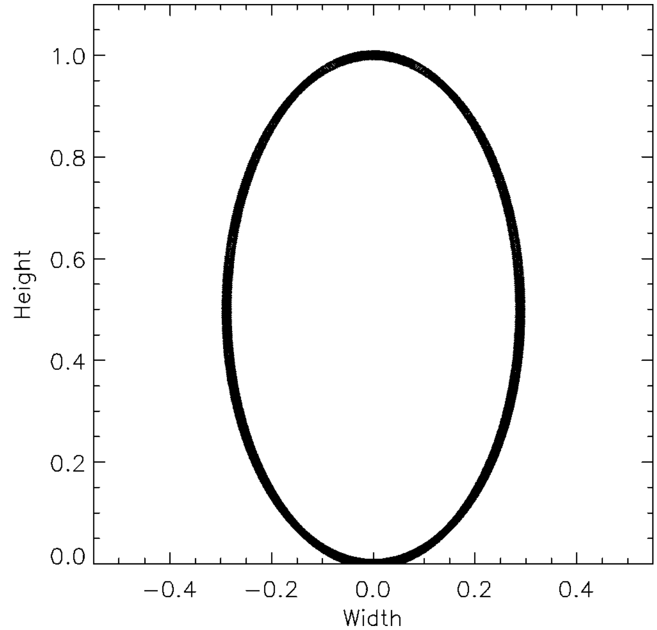

FIG. 2. Foil shape for deuteron recoils. Although the entire ellipse contributes recoils, all the way down to zero energy, which arrive at the same time, practically speaking, only the top portion of the ellipse is useful.

III. CALCULATION OF SIGNAL FROM A DEUTERON-RECOIL ELLIPSOID

The function of the shaped foil is thus to increase the surface area for producing recoils without compromising the time-of-flight dispersion originally present in the neutrons. Calculating the total signal produced requires integrating over all the angles over which recoils are generated, taking into account the differential scattering cross section in conjunction with the geometry of the foil.

Consider a small portion of the foil volume, dV. From this volume, the detector subtends a solid angle $d\Omega_{\text{det}}$. If the fluence of neutrons is I, the number density of recoil particles is n_r, and the differential scattering cross section in the laboratory frame is $d\sigma/d\Omega_{\text{lab}}$, then the number of scattered recoils, dN, produced within this differential volume that reach the detector is given by

$$dN = n_r I\, dV \frac{d\sigma}{d\Omega_{\text{lab}}} d\Omega_{\text{det}}. \tag{4}$$

Now, if the detector has a flat surface of area A_D, normal to the incident neutron direction, then

$$d\Omega_{\text{det}} = \frac{A_D \cos\theta}{r^2}, \tag{5}$$

where r is the distance from the detector to dV. The $\cos\theta$ term takes into account the projection of the area A_D in the direction of the foil volume element. The volume element dV is defined in the spherical coordinate system centered at the detector and is given by $dV = r^2 \sin\theta\, d\theta\, d\phi\, dr$.

Now, if the foil has a thickness, δ, which is very small compared to the scale of the entire foil, then, by using the shape of the ellipsoid, as given by Eq. (3), it can be shown that

$$\frac{dr}{\delta} = \left(1 + \tan^2\theta \left(\frac{1+\zeta\cos^2\theta}{1-\zeta\cos^2\theta}\right)^2\right)^{1/2} = S(\theta), \tag{6}$$

where the shape factor $S(\theta)$ carries the information about foil geometry.

The differential cross section in the laboratory frame is related to the differential cross section in the center of mass frame by

$$\frac{d\sigma(\theta)}{d\Omega_{\text{lab}}} = 4\cos\theta \frac{d\sigma(\gamma)}{d\Omega_{\text{CM}}}, \tag{7}$$

where the scattering angle in the center of mass frame, γ, is simply twice that in the laboratory frame, or $\gamma = 2\theta$.

Using Eqs. (5)–(7) in Eq. (4), and integrating over $0<\phi<2\pi$ and $\theta_{\min}<\theta<\theta_{\max}$, gives the total number of recoils reaching the detector:

$$N_{\text{tot}}=n_r I A_D \delta \int_{\theta_{\min}}^{\theta_{\max}} 8\pi \cos^2\theta \sin\theta S(\theta) \frac{d\sigma}{d\Omega_{\text{CM}}} d\theta. \quad (8)$$

This allows the total number of recoils, for a given fluence of monoenergetic neutrons, to be determined, provided that $\theta_{\min}$ and $\theta_{\max}$ are specified. The values of $\theta_{\min}$ and $\theta_{\max}$ would be fixed by constructing only the portion of the foil ellipsoid between these angles.

To determine the practical values of $\theta_{\min}$ and $\theta_{\max}$, issues which must be considered are as follows. (1) The recoil yield is raised by increasing the difference between $\theta_{\min}$ and $\theta_{\max}$. (2) The spread in recoil energies is reduced by decreasing the difference between $\theta_{\min}$ and $\theta_{\max}$. (3) For large values of $\theta_{\max}$, low energy recoils can reach the detector; however, such low energy ions will incur significant energy losses while escaping from the foil. (4) A neutron shielding plug should be installed to block the immediate line-of-sight between the detector and the target.

To investigate factors (1) and (2), the recoil yield distribution with angle, $dN/d\theta$, and with energy, dN/dE, will be calculated. The angular distribution is determined simply by differentiating Eq. (8) with respect to θ, giving

$$\frac{dN}{d\theta} \sim \cos^2\theta \sin\theta S(\theta) \frac{d\sigma}{d\Omega_{\text{CM}}}. \quad (9)$$

The energy distribution is given by $dN/dE = (dN/d\theta)/(dE/d\theta)$. Now the energy of the recoils is determined from Eq. (2) and is given by

$$E=\frac{4A}{(1+A)^2} E_n \cos^2\theta. \quad (10)$$

Dividing Eq. (9) by the differential of Eq. (10) (with respect to θ) gives

$$\frac{dN}{dE} \sim S(\theta)\cos\theta \frac{d\sigma}{d\Omega_{\text{CM}}}. \quad (11)$$

Note that this is simply the well-known energy spectrum of knock-on deuterons (given by $d\sigma/d\Omega_{\text{CM}}$) modified by the geometric factors $\cos\theta$ (the projection of the detector plane) and $S(\theta)$ (the shape factor of the foil).

For 14.1 MeV neutrons, $dN/d\theta$ and dN/dE are shown in Fig. 3. From these distributions, it appears that a reasonable value for $\theta_{\max}$ is given naturally by the low energy dip in the cross section at $\sim$10 MeV, corresponding to an angle of $\sim 25°$. This means, if only deuterons scattered between $0° < \theta < 25°$ reach the detector, they will have a relatively narrow energy spread between 10 and 12.5 MeV (neglecting energy losses incurred during escape from the foil).

The calculation of total recoil yields can now be performed. For $0° < \theta < 25°$, the integral in Eq. (8) (which is the effective scattering cross section) is 0.135 barns.

The scattering foil material used in this calculation is deuterated polyethylene (CD_2), which has a deuteron number density of 8×10^{22} cm^{-3}. The foil thickness is limited to $\delta=50$ μm since a 10 MeV deuteron would lose approximately 0.5 MeV traversing this distance. A 14.1 MeV neutron yield of 10^{13} and a foil distance of 160 cm (the OMEGA target chamber radius) will be assumed.

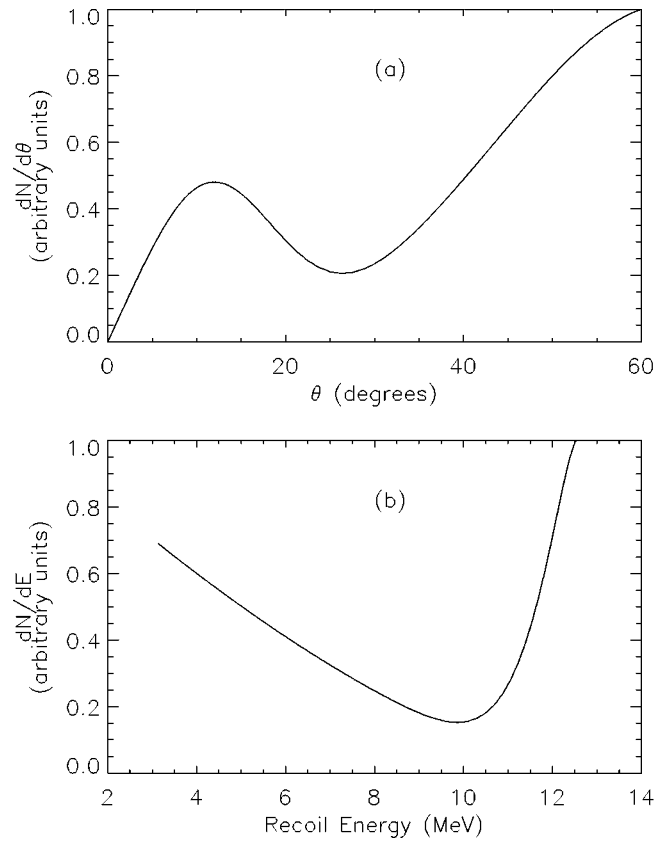

FIG. 3. Recoil deuteron angular and energy distributions after scattering by 14.1 MeV neutrons. (a) Differential angular yield of recoils for various angles as given by Eq. (9). (b) Energy distribution of recoils for scattering angles from 0° to 60°.

Using these values and a detector area of $A_D=1$ cm^2, in Eq. (8) gives a total recoil yield of 1700 recoils on a shot with a neutron yield of 10^{13}. The results are summarized in Table I. These numbers show that such a spectrometer can be used to measure the primary D-T yield and fuel ion temperature on current OMEGA experiments, mounting the detector and foil system outside the OMEGA chamber. Recoil spectrometers have the advantage over standard scintillators and activation counters in that they provide an absolute measure of neutron yield. Higher yields, such as those to be produced on the National Ignition Facility, will allow fuel ρR's to be

TABLE I. Results from a sample calculation to determine the number of deuteron recoils produced on a D-T shot with a neutron yield of 10^{13}. The parameters used are those which might be typical of an experiment on OMEGA.

D-T neutron yield	10^{13}
Foil distance from target	160 cm
Ellipsoid height ($2h$)	50 cm
Number density of deuterons in foil (CD_2)	8×10^{22} cm^{-3}
Foil thickness	50 μm
Detector area	1 cm^2
Accepted recoil scattering angles	0°–25°
Energy spread of recoils (no ranging)	10–12.5 MeV
Total No. of recoils	1700

determined from the secondary neutron spectrum.

The main source of noise will be direct neutron noise in the detector. Using the sample conditions outlined above, the number of direct neutron events is given by $N_n = I n_{\mathrm{det}} A_D \sigma_{\mathrm{det}} \Delta_{\mathrm{det}}$, where n_{det}, σ_{det}, and D_{det} are the number density, neutron cross section, and sensitive depth of the detector, respectively. Thus, the signal-to-noise ratio is given by $N/N_n = (n_r \sigma_{\mathrm{eff}} \delta)/(n_{\mathrm{det}} \sigma_{\mathrm{det}} \Delta_{\mathrm{det}})$, where σ_{eff} is the effective recoil scattering cross section calculated above — 0.135 barns for $0° < \theta < 25°$.

Now, using a silicon PIN diode, σ_{det} would be approximated by the total neutron interaction cross section for silicon ($\sim$1.5 barns). If the sensitive depth is $\sim$20 μm and $n_{\mathrm{det}} = 5 \times 10^{22}$ cm^{-3} (the number density for silicon), the signal-to-noise ratio would be $\sim$0.4. A second detector, without the scattering foil, would need to be run concurrently in order to generate a background spectrum which can be subtracted.

Signal-to-noise will be increased by utilizing a narrow, neutron shielding ''plug'' along the line-of-sight between the detector and the target, sacrificing a small number of forward scattered deuteron recoils. This plug would extend from the apex of the foil (at $\theta = 0$) out towards the target for several tens of cm. Note that the signal-to-noise can be improved further for higher energy neutrons since thicker foils (greater than the 50 μm used in this calculation) can be used — the higher energy deuterons being able to pass through more foil material with less energy loss. It is worthwhile examining the feasibility of constructing a large enough scattering foil to take advantage of the time delay between arrival of the faster direct neutrons and the slower deuteron recoils. With a sufficient delay to allow the detector to recover after the pulse of direct neutron events, the deuteron signal could be detected with much greater signal-to-noise.

The time response of such a recoil diagnostic is a function of the inherent time response of the detector (a silicon PIN diode), the bandwidth of the cabling system and recording instruments, the thickness of the scattering foil, the size of the detector, and particle statistics.[4] However, for the conditions given in Table I, the uncertainty in particle flight times caused by the 50 μm thick foil (and the resulting velocity losses) and the 1 cm^2 diode are each less than 1 ps. This means that, with sufficient particle statistics, the time response is dominated by that of the detector, or is approximately 50–100 ps using commercially available PIN diodes.

IV. CONCLUSION

The concept for a neutron time-of-flight spectrometer using a shaped scattering foil has been presented. Using a large area scattering foil in the shape of an ellipsoid increases the number of expected recoils above that of other proposed designs while maintaining the time-of-flight dispersion originally present in the neutrons. Such a design works only for recoils heavier than protons. Using a deuterated scattering foil, a D-T yield of 10^{13} will produce approximately 1700 deuteron recoils from 10 to 12.5 MeV, allowing measurements of primary yield as well as core ion temperature, on current experiments at OMEGA.

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